

# Example 1

1. *Calculate the solar zenith angle in Boulder at 3:00 pm on November 15.*
2. *Calculate the day length in Boulder on November 15.*
3. *Calculate the daily averaged TOA solar flux on November 15 in Boulder.*

1. The solar zenith angle is found from

$$\cos \theta_0 = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h$$

where  $\phi$  is the latitude,  $\delta$  is the solar declination, and  $h$  is the hour angle.

The solar declination is found from

$$\sin \delta = \sin \epsilon \sin \lambda$$

where  $\lambda$  is the longitude of the Earth from the vernal equinox. We'll assume the orbit is close enough to circular that we can use the linear in time approximation:

$$\lambda = 360^\circ \frac{t_v}{T}$$

where  $t_v$  is the time from the vernal equinox and  $T$  is the length of the year. November 15 is about 240 days since the vernal equinox, so

$$\lambda = 360(240/365.24) = 236.6^\circ$$

The Earth's tilt is  $\epsilon = 23.5$ , so the solar declination on November 15 is

$$\sin \delta = \sin(23.5^\circ) \sin(236.6^\circ) = -0.3327 \quad \delta = -19.4^\circ$$

The latitude of Boulder is  $\phi = 40^\circ$ . We'll assume the longitude is  $-105^\circ$  (center of the time zone) so that the hour angle is  $h = 360^\circ(3 \text{ hr}/24 \text{ hr}) = 45^\circ$ . Therefore, the solar zenith angle is

$$\cos \theta_0 = \sin(40) \sin(-19.4) + \cos(40) \cos(-19.4) \cos(45) = 0.297 \quad \theta_0 = 72.7^\circ$$

2. To find the day length we use the solar declination and the latitude:

$$\cos H = -\tan \phi \tan \delta = -(0.839)(-0.353) = 0.296$$

$$H = 1.27 \text{ rad} = 72.8^\circ. \text{ Day length is } (24 \text{ hr})H/180^\circ = 9.7 \text{ hrs}$$

3. November 15 is 49 days before perihelion, so the true anomaly is approximately

$$\nu \approx 360(-49/365.24) = -48.3^\circ$$

The ratio of the Earth-Sun distance to the mean distance is then

$$\frac{r}{r_0} \approx 1 - e \cos \nu = 1 - (0.017)(0.665) = 0.989$$

The daily averaged solar flux is

$$\bar{F} = \frac{S_0}{\pi} \left( \frac{r_0}{r} \right)^2 (H \sin \phi \sin \delta + \sin H \cos \phi \cos \delta)$$

Need  $H = 1.27$  in radians (but be careful with  $\sin H$ )

$$\bar{F} = S_0/\pi \left( \frac{r_0}{r} \right)^2 [1.27 \sin 40 \sin(-19.4) + \sin 72.8 \cos 40 \cos(-19.4)]$$

$$\bar{F} = S_0(0.989)^{-2}(-0.272 + 0.690)/\pi = S_0(1.022)(.133)$$

$$\bar{F} = (1366 \text{ W/m}^2)(0.136) = 186 \text{ W/m}^2$$